ISSN 0103-636X

Why is Mathematics Valuable? A Comparison of Turkish and German Mathematics Teachers

Por que a Matemática é Importante? Uma Comparação entre Professores de Matemática Turcos e Alemães

Yüksel Dede*

Abstract

This study was based on the data obtained from "Values in Mathematics Teaching in Turkey and Germany [VMTG]" project which lasted two years. The VMTG project compares German and Turkish mathematics teachers' and their students' values. The purpose of the study is to investigate Turkish and German mathematics teachers' views on why math is valuable, and explore the mathematical values behind their views. The study was undertaken with 9 Turkish and 13 German mathematics teachers. Even though several data collection instruments (i.e. observation sheets, Likert type (multiple choice) questionnaire, and questionnaire consisting of open-ended questions) were used in the project, only the data gathered from semi-structured interviews were the basis for the analysis of the study. Collected data were subjected to constant comparative method. Results revealed that two different major categories emerged: (1) isolated thinking and (2) connected thinking. Discussion, recommendation and further educational implications were provided at the end based on the data.

Keywords: Values. Mathematics Values. Turkish Mathematics Teachers. German Mathematics Teachers. Comparison.

Resumo

Este estudo é baseado nas informações obtidas no projeto "Values in Mathematics Teaching in Turkey and Germany [VMTG]" (Valores do ensino da Matemática na Turquia

* PhD in Mathematics Education at Gazi University (GU), Ankara, Turkey. Associate Professor of Mathematics Education at Istanbul Medeniyet University (IMU), Istanbul, Turkey. Mailing Adress: D-100 Karayolu, Merdivenkoy Mevkii No: 6/1, 34730 Goztepe- Kadikoy, Istanbul, Turkey. *E-mail*: ydede@medeniyet.edu.tr.

e Alemanha) feito nos últimos dois anos. O projeto VMTG compara professores de Matemática da Alemanha e da Turquia e os valores professados por seus alunos. O propósito do estudo é investigar o ponto de vista dos professores turcos e alemães sobre porque a Matemática é importante, e explorar os valores da Matemática que subsidiam suas opiniões. O estudo foi feito com professores de Matemática, sendo 9 turcos e 13 alemães. Mesmo sendo utilizadas várias formas de coleta de dados na pesquisa (ex.: observação de relatórios, questionátio de múltipla escolha e formulários de respostas abertas), apenas as informações obtidas em entrevistas semi estruturadas foram usadas como base de análise para este estudo. Os dados coletados foram submetidos a um constante método comparativo. Os resultados revelaram que duas grandes e distintas categorias surgiram: (1) o pensamento isolado e (2) o pensamento conectado. Ao final, seguem uma discussão, algumas recomendações e outras implicações educativas baseada nos resultados.

Palavras-Chave: Valores. Valores da Matemática. Professores de Matemática alemães. Professores de Matemática turcos. Comparação.

1 Introduction

This research is reported on the specific results of a wider comparative project concerning Turkish and German mathematics teachers' values. A short overview the goals of the project is presented below: i) To compare the Turkish and German mathematics teachers' values towards mathematics and mathematics education. As independent variables of the research, the teacher status, the type of school, the teacher's gender, and the experience of the teacher are considered. For this purpose, qualitative and quantitative research methods were used together (STAKE, 1995; YIN, 2003). For this, a questionnaire was developed to identify mathematics teachers' mathematics education values (DEDE, 2011). It was used to determine mathematics teachers who were willing to participate in the study and whose views regarding mathematics and mathematics education values. A detailed interview protocol was also prepared to determine math teachers' the mathematics and mathematics education values. This protocol was developed by the researcher; then, it was presented to three academicians with PhD degrees respectively in the fields of mathematics education, science education and educational sciences to consider for the assessment and comment. ii) To investigate which mathematics educational values the teachers actually convey to their mathematics teaching via classroom observation. This component of the research is included in order to provide footing enactive to the espoused values. For this goal, observations of lessons had conducted with mathematics teachers from Turkey and Germany (which are selected from the interviews) to have the detailed and in-depth information. iii). To investigate to what extent teachers can transfer the mathematical values that they have and convey to the classroom environments. For this aim, semi-structured interviews have conducted with students from Turkey and Germany (selected amongst the students of the teacher to whom are interviewed) to have detailed and in-depth information. However, this study will only focus on Turkish and German mathematics teachers' views on why math is valuable, and explore the mathematical values behind their views.

1.1 Values and Teacher Values

Up to the middle of twentieth century, the concepts of value-free culture and value-free education were mostly popular. Those days were dominated by the positive beliefs which did not attach any specific social value systems, were based on objective, rational and empirical criteria and established as a result of technological developments and scientific discoveries. Old theories ignored the importance of moral factors. These theories evaluated the effects on these changes in the light of technological developments instead of viewing the social changes as the results of moral decision of social action. But this trend of thinking has slowly changed with the understanding of the educational and socio-cultural changes and it has also begun to contain the value orientations (LEE, 2001). Values are general guide for the behavior emerging from one's experiences and relations in his/her life (RATHS; HARMIN; SIMON, 1987). According to this, values play a role on one's behaviors, decisions and selections consciously or unconsciously (FITZSIMONS; BISHOP; SEAH; CLARKSON, 2001). For this reason, the transmission of values and culture is one of the aims of the education. Schools are key institutions where this function is realized and sustained (OSLER; STARKEY, 2001). Therefore, values appear in school's goals, activities, and curricula as well as in the requirements set by the state. Value-free education is unlikely in most places (POWE, 1993). For example, in Turkish, high school mathematics curriculum (Ministry of National Education [MEB], 2005, p. 1), the first general goal of Turkish National Education is stated as, "[...] to grow [students] as citizens who embrace, protect and improve the Turkish nation's national, ethical, humanitarian, spiritual and cultural values [...]". On the other hand, the values have an impact on teachers' decision and behavior (FASHEH, 1982). Besides, teachers' decision making skills are associated with their prior experience, values and beliefs, teaching aims and objectives, decision schema, teaching situation, and decision and action (BISHOP; WHITFIELD, 1972). Gudmunsdottir (1990) viewed the values as the guide of teachers' practices. Chin and Lin (2001) viewed the values as personal decisions related to individual standards for opinions and behavior which are considered as worthwhile and important. Thus, values can be perceived as teachers' pedagogical identity and viewed as decisions and judgment which are seen as important or valuable according to their pedagogical identity (CHIN; LIN, 2000).

1.2 Mathematics Philosophy and Values

Mathematics is perceived as an abstract, cold and inhuman study in the large societies (ERNEST, 1998). According to Wong (2005, p.1), the prevailing public image of mathematics is an objective, abstract, and inhuman subject. Mathematicians are often perceived to be human beings born with special talents in logical reasoning and skilful manipulation of arcane symbols.

For that reason, it is related to absolutist philosophies (ERNEST, 1998), because absolutist philosophers claim that the structure and objects of mathematics existed previously regardless of human being, and thanks to this, mathematical knowledge consists of connections which combines these structures and objects. According to absolutist philosophers, mathematical knowledge which emerges as a result of human endeavor should be considered among subjects like psychology and sociology rather than mathematical philosophy (BAKI, 2008). Accordingly, mathematics is a profession and detached from values; that is, mathematics is value-free and culture-free (ERNEST, 1998). To them, how can indisputable concepts such as fraction, triangle and multiplication be related to values (BISHOP, 1988)? In this point, the expressions of Ernest (2007) given below are interesting:

> The traditional foundationalist schools of formalism, logicism and intuitionism sought to establish the absolute validity of mathematical knowledge by setting-up foundational systems. Although modern philosophy of mathematics has in part moved away from this dogma of absolutism, it is still very influential, and needs to be appraised (p.174).

On the other hand, fallibilist philosophers opposed to this view of the absolutist philosophers and indicated that mathematics was consistent with "connected" values (ERNEST, 1998, p.45). According to fallibilist philosophers, fallibility has a crucially important place in the construction and development of mathematical knowledge. They claim that human work and history play an important role in knowledge construction, and mathematical knowledge is an output of a process of human work and social interaction. For that reason, mathematical knowledge should be considered within social sciences (BAKI, 2008). However, they did not reject the role of mathematical structure, but refused the view that mathematics supports the unique, fixed and continuous hierarchical structure. In addition, contrary to absolutist philosophers, they also claim that mathematics is value-laden and culture-laden (ERNEST, 1998). In the literature, it is noted that there are several views which support this claim (BISHOP, 1988; ERNEST, 1991; LIM; ERNEST, 1997; BISHOP, 1998a; BISHOP, 1998b; CLARKSON; BISHOP, 1999; BISHOP, 2002; ERNEST, 2007).

1.3 Mathematics Pedagogy, Culture, and Values

Education in general and mathematics education in particular portray the values actively and transfer these values (SEAH; BISHOP, 2002; SEAH, 2003). Hence, two different viewpoints related to mathematical philosophy given above have different effects on classroom practices (ERNEST, 1991). For example, while a math teacher who adopts a fallibilist view considers the problem-based teaching or practice more valuable for his/her instruction (SEAH, 2003), the other teacher who adopts an absolutist view considers teacher-centered or deductive approach more valuable. In addition, according to Seah (2002), values are crucially important parts of math learning and teaching. Clarkson and Bishop (1999) also supported a similar approach. However, in spite of this, they indicated that the importance of values is not very well known by mathematics teachers. Clarkson and Presmeg (2008) showed that when mathematics teachers are asked "which values are you teaching in your teaching?" even today, they replied "None. I'm teaching mathematics" (p.229).

The research on values in math teaching appeared in 1980s by integrating them to cultural dimension of math education (BISHOP, 2004). Bishop (1996) classified three types of values observed in the mathematics classrooms. They are general educational, mathematical, and mathematics educational values. Bishop (1998b) indicated them as general educational values, honesty and good behavior. Bishop (2004) described three pairs of complementary mathematical values in the Western culture as (1) rationalism and objectism, (2) control and progress, and (3) openness and mystery. On the other hand, Bishop (2004) also conceptualized mathematics educational values as being formalistic view and activist view, instrumental understanding and relational understanding, relevance and theoretical knowledge, accessibility and specially evaluating and reasoning (For further details, see BISHOP, 1998b, 2004). According to this, educational values are related to general societal values, mathematical values are related to the scientific discipline of mathematics, and mathematics educational values are related to pedagogy of mathematics that is, to the practices and norms emerging from mathematics instruction (SEAH; BISHOP, 1999; ATWEH; SEAH, 2008). In addition, these three categories of values are not independent from one another. Therefore, some values may be related to two or three of these categories at the same time. For example, the values of progress and creativity are both related to mathematical, mathematics educational and general educational values (SEAH; BISHOP, 2000; SEAH, 2008). Moreover, mathematics is a culture (BUTTY, 2001) or "mathematics… conceived of as a cultural product, which has developed as a result of various activities" (BISHOP, 1988, p.182). National Council of Teachers of Mathematics (NCTM) (2000) also views mathematics as a part of cultural heritage and indicates the value of this point: "Mathematics is one of the greatest cultural and intellectual achievements of human-kind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects" (p.4).

So, the approaches towards mathematical studies vary in terms of communities, cultures and time (LANCASTER, 2006). Seah (2003) suggested that cultural differences influence how the same mathematical content can be taught with different approaches and that different cultures affect the associated values. That is, cultures do not share the same values (BISHOP; CLARKSON; FITZSIMONS; SEAH, 2000). Seah (2005) stated:

> Values related to mathematics education are inculcated through the nature of mathematics and through the individual's experience in the socio-cultural environment and in the mathematics classroom. […] They also influence the development of other affective constructs related to mathematics education and to life (p.146-147).

In this respect, since mathematics teaching varies in different culture and educational system, the math educational values may also vary. For example, while the "technology" as a math education value is important in one education system, it may not be perceived as important in another educational system (ATWEH; SEAH, 2008).

1.4 Mathematics Curriculum and Teaching in Turkey and Germany

There are some differences in the content of mathematic curriculum in Turkey and Germany. Ministry of National Education (MEB) for Elementary and Secondary Education, and Higher Education Council (YÖK) for Higher education in Turkey are responsible bodies for planning, implementation and coordination. The national and international comparative studies on mathematics teaching indicated that Turkish students' mathematics achievement were observed to be lower than the students' mathematics achievement in other countries (Education Research and Development Directorate – EARGED, 2005). In order to understand this problem, primary school and secondary school mathematics curricula were newly developed on the basis of constructivist approaches in Turkey. These new curricula were re-designed by considering correct, effective and good Turkish speaking, and the skills regarding critical thinking, creative thinking, decision making, and technology usage as well as four mathematics skills of problem solving, connections, communication and reasoning (MEB, 2009a). Then, accreditation of faculties of education based on constructivism was started in 1997 and continued in successive years till 2007. Curriculum and the standards of all teacher education programmes are planned by the joint cooperation of MEB and YÖK (EURYDICE, 2010a) and preservice mathematics teachers take courses related to mathematics content knowledge, pedagogical knowledge and general culture.

On the other hand, even though the implementation of education varies among the 16 states in Germany, the Standing Conference (Kultusministerkonferenz) coordinates Ministry of National Education's works in each state (RILEY; MCGUIRE; CONATY; DORFMAN, 1999). In the international studies, German students obtained near to international average score in the field of literacy, math and natural sciences. However, these scores were observed to be very poor when compared with some Asian and European countries (MISEK, 2007; SCHUMANN, 2000). Teacher education in Germany varies in states. Responsibility for teacher training rests with the Ministries of Education and Cultural Affairs of the *Länder* which regulate training through study regulations and examination regulations (EURYDICE, 2010b, p.25). Mathematics teacher candidates take courses in mathematics, education and social studies, didactics of mathematics and the second school subjects (KEITEL, 1992). However, while Germany signed the Bologna Reform agreement in 1999, Turkey signed it in 2001. Therefore, Bachelors and Masters Systems are now updated based on the standards of the Bolognaized university system in both countries.

2 Scope and Purpose of the Study

Cross-national studies not only provide data on diagnosing and decision making of how students' learning occur but shed also light on the issues on math learning and teaching (CAI, 2006). In this regard, this study reports Turkish and German mathematics teachers' mathematics values. The causes behind the choice of these two countries are as follows: i) Teachers in the two countries play an important role in mathematics learning and teaching; ii) The Federal Republic of Germany represents Western, liberal culture and has a multicultural society. Turkey is different from Germany in some aspects, in particular culture, language, and religion. Turkey is a bridge between Western and Eastern cultures and has taken series steps for Westernization. It is suggested that cultural differences may influence how the same mathematical content can be taught through different approaches and that different cultures affect the associated values (SEAH, 2003); iii) It is thought that this comparison may provide a significant contribution to the literature and discussion concerning which values may be transferred to immigrating students with foreign nationalities, particularly Turkish students in Germany, because students in Germany come from a multicultural background, whereas Turkish students come from a homogeneous background; iv) When the literature on values in math education was searched, no study was encountered investigating a comparison of Turkish and German mathematics teachers in general, or on their values in particular.

On the other hand, studies such as the Values in Mathematics Teaching (VIMT), and the Values and Mathematics Project (VAMP) have demonstrated that the role of values and their importance are situated and also content-specific in mathematics education (LEU; WU, 2000). These studies have shown the role played by values in mathematics education and instruction; but there are not many studies focused on finding out or measuring in-service teachers' mathematics-related values, which influence a person's choices and behavior (YERO, 2002), and "standards that guide ongoing activities" (ROKEACH, 1973, p.12). Values influence teachers' decisions and behaviors (FASHEH, 1982);

therefore, the causes of the behavior and the classroom practices and preferences can be best understood through examining teachers' mathematics-related values. In this sense, this study reported on the specific findings of a wider comparative study concerning Turkish and German mathematics teachers' values. The purpose of this study was to investigate the views of Turkish and German mathematics teachers on why math is valuable and explore the corresponding values to their views. Specifically, the study attempted to address to following questions:

1. Why is mathematics valuable to Turkish and German math teachers?
2. How do these values differ from Turkish math teachers to German math teachers?

3 Method

3.1 Research and Design

Designed as mixed method, this study was based on the data obtained from VMTG project which lasted two years. In this study, some parts of the data collected through semi-structured interviews were reported. For this reason, the present study was only limited with data qualitative in nature. Due to the nature of the values, quantitative approaches require subjective and arguable understanding. For this reason, the studies focusing on values in mathematics education have mainly been designed as qualitatively (SEAH, 2008). Moreover, the mathematics curriculum includes both explicit and implicit values. Therefore, implicit values were presented in a hidden manner, acquired in more subtle ways, and evidenced in the learner's behavior. The explicit values were planned explicitly, applied in the classrooms, and acquired from the instruction. In the current study, the explicit values stated by the teachers and to be acquired by learners have been documented using semi-structured interviews and fields notes, whereas implicit values not considered in the study would need to be based on more inferential data sources such as classroom observations (DEDE, 2011). Therefore, the definition of value used in this current study can be considered as personal preferences for stating if a thought and statement are of worthwhile and importance for the individual (CHIN; LIN, 2001; SEAH, 2002; SWADENER; SOEDJADI, 1988).

3.2 Subjects

In this study, the teachers were selected based on a combination of purposeful and theoretical sampling methods. Purposeful sampling method was used as a means of selecting information rich cases for this investigation (PATTON, 1990). The maximum variation sample is a special kind of purposeful sampling method and, in this investigation, it was used to capture mathematics teachers' values from two different nationalities. On the other hand, theoretical sampling method was used to reach saturation point and was guided by the emerging categories in this study (STRAUSS; CORBIN, 1998) and the process of theoretical sampling was combined with the constant comparative method (KOLB, 2012). So, twenty- two mathematics teachers were selected for semi-structured interviews. They were thirteen German and nine Turkish mathematics teachers. All of the German mathematics teachers were selected from primary and secondary schools in Berlin whereas seven of Turkish mathematics teachers were from primary and secondary schools in Sivas and two in Ankara. All German mathematics teachers were authorized to teach math up to 10^{th} grade and six of them were authorized to teach math up to 13^{th} grade. Of Turkish math teachers, four were authorized to teach up to 12^{th} grade whereas five were authorized to teach up to 8^{th} grade. (Compulsory education in Turkey is 8 years, and secondary education lasted four years. Elementary and secondary education in Germany differs according to the states. Elementary education is 6 years while secondary education is 7 years in Berlin). One of the selected teachers in both countries had a leadership role in their school and school district, and attended in-service training as trainers. Two of German teachers obtained master degree in mathematics education and one did PhD in theology. Two of Turkish teachers obtained master degree on educational sciences. Eleven of German mathematics teachers were females and two were males. One of Turkish mathematics teachers was female and eight were males. Female teachers who had been invited to participated in this study were rejected the invitation. The teaching experience of German mathematics teachers was ranged from 1 to 30 years whereas that of Turkish math teachers was ranged 4 to 30 years. Six of Turkish teachers graduated in 1997 from the Faculty of Education, Department of Mathematics Education whose curricula were revised and re-designed based on constructivist approaches.

3.3 Data Collection

Even though several data collection instruments (i.e. observation sheets, Likert type questionnaire-multiple choice, and questionnaire consist of open-ended questions) were used in the project of VMTG, only the data gathered from semi-structured interviews were the basis for the analysis of the study.

3.3.1 Semi-Structured Interviews

3.3.1.1 Preparation of the interview protocol

A detailed interview protocol was prepared to clearly determine teachers' mathematical values. This protocol was developed by the researcher, then, it was presented to three academicians who had PhD degree in the field of mathematics education, science education and educational sciences in order to obtain expert opinion. These three academicians are experts especially in the qualitative research. Also, the research topic of the academician having PhD degree in the field of educational sciences is on value education. Final form of the interview protocol was given based on the suggestions taken form the experts.

3.3.1.2 Conducting the interviews

Interviews were undertaken between the years of 2008 and 2010 in Ankara and Sivas, Turkey and in Berlin, Germany. Twenty-two in-depth semi-structured interviews with math teachers were conducted and analyzed. Each mathematics teacher was clearly explained the purpose of interviews before the start. Interviews were conducted in a comfortable and an appropriate atmosphere by the author. The interviews were audio taped with getting the permission of each interviewee. Only one German mathematics teachers did not give permission to be audio taped. For this reason, an interview with this teacher was written on a separate sheet. In addition, studying in one of German Universities, a student who spoke both Turkish and Germany in advanced level helped the researcher in the interviews undertaken with German teachers. The names of the participants were kept anonymous for ensuring the confidentiality. For this reason, codes such as G1,…, G13 for German teachers and T1,…, T9 for Turkish teachers were used. Each interview lasted about 50-110 minutes. The questions and examples differed according to the teacher's responses.

3.3.1.3 Translation of Semi-Structured Interviews

The interviews were conducted with Turkish teachers in Turkish and with German teachers in German. With this way, teachers were given an opportunity to express their views comfortably without having any language problems. The interviews with German teachers were firstly translated by the researcher into Turkish. Same interviews were also independently translated into Turkish by the student mentioned above. In addition to these translations, same interviews were again independently translated into Turkish by one Turkish teacher who had an advanced language skill in German and Turkish. This teacher graduated from the department of German Language and Literature from two universities, each in Turkey and Germany, and worked in private school in Germany. After the translation of the interview by three independent translators, the translated documents were compared with regard to differences and similarities, and thus agreement was reached at the end.

3.4 Data Analysis

In this study, it was only focused on the analysis of the responses of teachers to the questions of “Is math valuable to you? Why?” The analysis of the data collected in the study was continued until the saturation was reached (ARBER, 1993). In this way, definition, explanation and classification of the data were done. Sub-categories were emerged when at least half of the teachers are common responses. Semi structured interviews with teachers were analyzed through making use of constant comparative method (CCT). The types of “comparison of interviews from groups with different perspectives but involved with the subject under study” (BOEIJE, 2002, p.396) of CCT was used. CCT consist of there steps; (1) open, (2) axial and (3) selective coding (GLASER; STRAUSS, 1967; STRAUSS; CORBIN, 1998). Open coding is realized with starting category of the information on the phenomenon under investigation, and segment information (CRESWELL, 2008). With this way, the meaning and thinking of the concepts are revealed, and units are identified based on text and the topic of the research. After that, these units are divided into categories and sub-categories (PITNEY; PARKER, 2002). For this reason, the researcher tried to understand the meaning of the data (interview transcripts) by reading more than one without considering any theory, and then coded them. At the end of the coding, it was observed that 54 open coding for German teachers and thirtythree open coding for Turkish teachers emerged. Some of the coding examples were “a course used in real life”, “related to common culture”, “used for understanding the earth”, and “teach logical thinking”. In the step of axial coding, categories and sub-categories were established for the certain and complementary explanations regarding the phenomenon under investigation (PITNEY; PARKER, 2002). In the present study, in this step, twenty-six sub-categories for German teachers and fifteen sub-categories for Turkish teachers were established by the researcher. At the end of the coding, two main categories were constructed. For example, for German teachers, category of “isolated thinking” consisted of twelve sub-categories such as “logical thinking”, “structural”, and “abstract”. In the step of selective coding, the relationship of categories with sub-categories and further with other data is sought. In this step, a central category which explained the phenomenon and associated with all categories was developed. Central category can be emerged from one of the categories or the conceptual content of the other categories (PITNEY; PARKER, 2002). In the present study, central category was developed by using the conceptual content of emerged two categories. Thus, central category was named as “the values regarding the static and dynamic aspects of math”.

3.5 Trustworthiness of the Study

The research data were collected by semi-structured interviews. In order to categorize the data obtained from the interviews and determine the common expressions, the interview transcripts of the teachers were read more than one. The words used by the teachers were exactly written down on the paper without making any changes. Then these written texts were given to the teachers for their approvals which enabled the member checking for the reliability of the data. Because, according to Creswell (1998), a member check requires “taking data, analyses, interpretations, and conclusions back to the participants so that they can judge the accuracy and credibility of the account” (p.203). Peer review or debriefing process was used for the confirmation of the reliability of the research data. This process is an external control tool for the reliability of the research (LINCOLN; GUBA, 1985). Interviews’ transcripts of this section were about 40 pages. They were written on paper as average length of two pages and categorized. This categorization process was firstly done independently by the author and the experts above mentioned. Afterwards, they were compared and adjusted after reaching a consensus by the experts.

4 Findings

4.1 A Brief Data

Totally, thirty- three different themes emerged from the responses to semi-structured interviews. Description of each sub-category is summarized in Table 1.

Table 1: Comparison of the teachers' mathematical values associated with sub-categories

Teacher			
German		Turkish	
Sub-category	Corresponding values	Sub-category	Corresponding values
1. Usefulness in the real world	Relevance, Usefulness	1. Usefulness in the real world	Relevance, Usefulness
2. Intellectual activity	Rationalism, Mystery, Mental Development	2. Intellectual activity	Rationalism, Mental Development
3. Logical thinking	Rationalism, Reasoning	3. Logical thinking	Rationalism, Reasoning
4. Language	Communication, Tool, Objectivism	4. Terminology of a language	Communication,Tool, Objectivism
5. Flexibility in thinking	Progress, Flexibility	5. Flexibility in thinking	Progress, Flexibility
6. Esthetics	Esthetics	6. Esthetics	Esthetics
7. Understand real-world	Tool, Control	7. Understand real-world	Tool, Control
8. Applicability	Applicability	8. Applicability	Applicability
9. Culture	Communication	9. Calculation	Rationalism
10. Generalization	Rationalism, Process	10. Intelligence indicator	Rationalism, Tool
11. Regularly	Rationalism	11. Foundation for science, technology and so on.	Tool, Control, Applicability
12. Combination	Tool	12. Money	Product, Tool
13. Systematic	Rationalism	13. Economical	Economical
14. Cognition	Rationalism, Process	14. Consistent	Consistent
15. Problem solving	Process, Progress	15. A path to God	Tool
16. Composition	Tool, Progress		
17. Structural	Rationalism		
18. Clear scientific	Clarifying, Openness		
19. Creativity	Progress, Creativity, Mystery		
20. Game	Fun, Tool		
21. Abstract	Rationalism		
22. Classification	Tool, Process		
23. Historical Background	Process		
24. Accurate	Rationalism		
25. Precise	Rationalism		

These themes were sub-categorized into two major areas for further reflection: isolated thinking and connected thinking. Details on these categories are given below:

4.1.1 Isolated Thinking

This category reflects the common image that mathematics is abstract, inhumane and objective subject (ERNEST, 2004). In other words, this category encourages the protection of the essence (WANG; LIN; CHIN; CHANG, 2006) and nature of math and putting more emphasizes on this point in mathematics. In addition, it is pointed that rule, abstractness, formalism, logicism, rationalism, absolutism and deduction in mathematics are more important and valuable. That is, these values see the mathematics as "separate knowing" (OCEAN, 2005, p.137). This category can be grouped within the two (platonism and formalism) of four important schools of thinking (MALVERN, 2004). Put another way, it is indicated that mathematics is value-free and reflects the views of absolutist philosophers (ERNEST, 1998; 2007).

4.1.2 Connected Thinking

This category reflects the socio-cultural dimensions of mathematics, and the mathematical thinking that is influenced by these dimensions (BISHOP, 2008). Also, this category presents that human studies and history are important factors for building mathematical knowledge (BAKI, 2008). In short, this category points that mathematics include connected values (ERNEST, 1998, p.45). In other words, this category emphasized that usefulness, process, intuitiveness, communication, creativity, tool, tolerant models, induction and relativity are more important and valuable in mathematics. This category can be grouped within two (intuitionalism and conceptualism) out of four important schools of thinking (MALVERN, 2004). That is, it is indicated that mathematics is value-laden and culture-laden and reflects the views of fallibilist philosophers (ERNEST, 1998, 2007). Shortly, they see the mathematics as "connected knowing" (OCEAN, 2005, p.137).

4.2 The Germany Data

4.2.1 Isolated Thinking

As the sub-categories presented in Table 1 examined, German mathematics teachers' responses related to the category of "*Isolated Thinking*" can be grouped under eleven sub-categories; such as, intellectual activity (rationalism, mental development), logical thinking (rationalism, reasoning), generalization (rationalism, progress), regularly (rationalism), systematic (rationalism), cognition (rationalism, process), structural (rationalism), clear scientific (clarifying, openness), abstract (rationalism), accurate (rationalism), and precise (rationalism). Among others, rationalism as a mathematical values took over. Furthermore, the values of mental development, reasoning, progress, process, and openness were also emphasized. Some quotations from the interviews with two German mathematics teachers and the values corresponding to these quotations are given below

> G1: Yes, it is valuable for me. That is why I became a math teacher (laughing)... and (thinking) Why? Because,… mathematics is especially a shape of thinking. And [it is] very precise discipline in some degree and a logical way. I believe that [math] is one of the courses contributing and improving mental development. A little thing in the mathematics is even precisely accurate.

The values of cognition, rationalism, reasoning, and mental development took over with the statements of G1, "[...] a shape of thinking […]" "[...] very precise discipline […]", "[...] a logical way […]", "[…] one of the courses contributing and improving mental development […]", and "A little thing in the mathematics is even precisely accurate" (see sub-categories of 5, 25, 3, 2, and 24 in Table 1, respectively). In the following interview quotation of G8, the values of rationalism and reasoning were emphasized in the statement of "orienting to the systematic thinking, teaching logical thinking and encouraging to propose argument" (see sub-categories of 13 and 3 in Table 1, respectively).

> G8: I see the math as very important course. In my field, when I think free of myself, it is not I am math teacher (laughing). But, there are many things in it. Orienting to the systematic thinking, teaching logical thinking and encouraging to propose argument. These types of things are necessary for the person. Finally, they are in the general education.

4.2.2 Connected Thinking

As sub-categories in Table 1 are examined, it is seen that the responses of German mathematics teachers are grouped into fourteen sub-categories; such as, usefulness in the real world (relevance, usefulness) language (communication, tool), flexibility in thinking (progress, flexibility), aesthetics (esthetics), understanding real-world (tool, control), *c*ulture (communication, tool), applicability (applicability), combination (tool), problem solving (process, progress), composition (tool), creativity (creativity, intuitiveness, mystery), game (fun, enjoyment), classification (tool, process), historical background (process). Looking at these sub-categories, it is clear that German mathematics teachers gave more emphasis on the mathematical values of usefulness, communication, tool, flexibility, applicability, esthetics, process, creativity, and fun. Some quotations from the interviews with three German mathematics teachers and the sub-categories corresponding to these quotations are given below.

> G10: Yes, certainly (laught).Because, the models in mathematics can also be applied to other things. Mathematics is a very clear science. Everybody can understand its language. That is, it is too beyond normal languages. (thinking two seconds). Well, there is a historical foundation of it; it combines various societies and time periods.

G10, the statement of "the models in mathematics can also be applied to other things" correspond to the value of applicability; the statements of "Mathematics is a very clear science. Everybody can understand its language" correspond the value of openness; the statement of "there is a historical foundation of it, it combines various societies and time periods" correspond to the values of process and tool (see sub-categories 8, 18, and 23 in Table 1, respectively).

> G12: Yes, yes (laughing). It is a good opportunity to get intellectual active, it teaches abstract thinking. You can get the results through making tryouts. Mathematics has also role in creativity and aesthetic (after three seconds). Of course, it has also cultural value. As examining the history and past of the mathematics, it is perceived as a huge discovery by the people. Mathematics attracts the minds and inspires.

G12, the statement of "It is a good opportunity to get intellectual active" correspond to the value of "mental development"; the statement of "it teaches abstract thinking" correspond to the value of abstract; the statement of "Mathematics has also role in creativity and aesthetic" correspond to the values of creativity and aesthetic; the statement of "As examining the history and past of the mathematics, it is perceived as a huge discovery by the people" correspond to the value of process (see sub-categories 2, 21, 19, 6, and 23 in Table 1, respectively). G4 whose interview quotation is given below found mathematics valuables in 6 dimension. These are i) fun; ii) intellectual activity; iii) classification; iv) abstract; v) logical thinking, and vi) usefulness in real life (see sub-categories 20, 2, 22, 21, 3, and 1 in Table 1, respectively). The values of fun, classification, and usefulness in daily life are related to "connected thinking", whereas the others to "isolated thinking".

> I: Is mathematics valuable for you?
> G4: Yes, it is so valuable
> I: Why?
> G4: It gives me fun.
> I: Why is it funny?
> G4: What gives me fun is keeping me busy with something, and dealing with something that forces me. Well… [it] is a [piece of] work that shows intellectual hardness. And this is also funny for me; listening to certain things, subtracting and so on.And saying unimportant things to all other than these. Apart from these unimportant things, solving that problem, I also find fun in this.
> I: What else is mathematics valuable?
> G4: It helps me so much in my life. And, well…because…because this abstract thinking problems, mathematical logical thinking has helped me so much in my life and in my university life. And also at school and also out of it.I am thinking that [it] makes many things so easy and simple.

4.3 The Turkey Data

4.3.1 Isolated Thinking

Turkish mathematics teachers responses on this category can be grouped into 4 sub-categories: intellectual activity (rationalism, mystery, mental development), logical thinking (rationalism, reasoning), calculation (rationalism), and intelligence indicator (rationalism, tool). These sub-categories show that Turkish mathematics teachers generally hold the value of rationalism. In addition, they put emphasis on the values of mental development and reasoning. Some quotations from the interviews with two Turkish mathematics teachers and the sub-categories corresponding to these quotations are given below:

> T7: Mathematics is for me in reality, other teachers also say that Mr X (name is not given), "Mathematics course is enough, [he] does not see another course which is enough". I see the mathematics and intelligent equally[…] thinking and judgement require intelligent. For this reason, mathematics goes one step further.

> T2: […] Mathematics is a course that develops the intelligence. To me, this is a purpose of mathematics.

> Teaching of different thinking, analyzing, synthesizing, and reaching the results based on the reasons are the purposes. I believe that the purposes of the mathematics are those.

In the quotation of G7 given above, G7 saw the mathematics as an intelligent indicator, which is a tool, with the statement of "I see the mathematics and intelligent equal… thinking and judgement require intelligent." (see sub-category 10 in Table 1). T2 saw the mathematics as tool and more emphasized the mathematics values of mental development, flexibility and rationalism with the statement of "Mathematics is a course that develops the intelligent. Teaching of different thinking, analyzing, synthesizing" (see sub-categories 2, 5, and 14 in Table 1, respectively).

4.3.2 Connected Thinking

Turkish mathematics teachers responses on this category can be grouped into eleven sub-categories; usefulness in the real world (relevance, usefulness) terminology of a language (communication, tool, objectivism), flexibility in thinking (progress, flexibility), aesthetics (aesthetics), foundation for science, technology and so on (tool, control), understand real-world (tool, control), money (product, tool), economical (economical), applicability (applicability), consistent (consistent), and a path to God (tool) (the statements given in the paranthesis show the values corresponding to that category). These sub-categories indicate that Turkish mathematics teachers put more emphasis on the mathematics educational values such as usefulness, communication, flexibility, aesthetics, tool, economical, product, consistent, and applicability. Some quotations from the interviews with three Turkish mathematics teachers and the sub-categories corresponding to these quotations are given below:

> T5: Mathematics is important for me. Because… (thought in three seconds) mathematics is a part of my life that are my experiences now. It is not only in the school, but also in bazar, in doing something, in going to somewhere from somewhere, in another organization... I am benefiting from the logic of the mathematics, mathematical logic, the thing of thinking, as far as I understand.

Based on the quotation given above, T5 gave importance to the mathematics values of usefulness and relevance with the statement of "It is not only in the school, but also in bazar, in doing something, in going to somewhere from somewhere […]" (see sub-category 1 in Table 1).

> T1: […] mathematics provides easiness in all aspects of life to you; we know that as a science, mathematics is in the base of exact science. So, in all sciences, if an aesthetic view point is provided in art, if [it] gives a chance to interpret the different view… well… in philosophy, if it is entering the philosophy as the subject of logic, if your are an architect and see the mathematics in your product and it gives us pleasure, well… if [it] makes our work easy, […] when I understand the contributions to all these different subjects in the society, it comes to me as a very nice course.

The quotation taken from the interview with T1 showed that this mathematics teacher put more emphasis on the mathematical values of tool and control with the statement of "we know that as a science, mathematics is in the base of exact sciences"; on the mathematics value of aesthetic with the statement of "if aesthetic view point is provided in art, […] if you are an architect and see the mathematics in your product"; on the mathematics value of flexibility with the statement of "if [it] gives a chance to interpret the different view"; on the mathematics value of applicability with the statement of "if it is entering the philosophy as the subject of logic" (see sub-categories 11, 6, 5, and 8 in Table 1, respectively). T4 whose interview quotation is given below saw the mathematics as tool and products with the statement of "First of all, I gain my income from this subject…if given to me, I may not deal with mathematics" (see sub-category 12 in Table 1).

> T4: Mathematics is valuable for me. First of all, I gain my income from this subject. Mathematics is in our education system, if given to me, I may not deal with mathematics. I became mathematics teacher after a long time.

4.4 Comparison between Turkey and Germany

The results revealed that there were similarities and differences among Turkish and German mathematics teachers' responses and values regarding to why mathematics is valuable. These similarities and differences are as below:

4.4.1 Similarities

As presented in Table 1, first eight sub-categories are common for both groups of teachers. Common sub-categories are; Usefulness in the real world, intellectual activity, logical thinking, language, flexibility in thinking, esthetics, understanding the real world, and applicability. The values crossing them, respectively, are: Usefulness, rationalism, rationalism (reasoning), communication, flexibility, esthetics, tool, and applicability (see Table 1 for other values). Out of these values, the value of rationalism pertains to mathematical value whereas the values of usefulness, communication, flexibility, aesthetics, tool, and applicability pertains to mathematics educational values. On the other hand, only two of these sub-categories (intellectual activity and logical thinking) are within the category of "isolated thinking" whereas the others are within the category of "connected thinking". This reflects that mathematics teacher in both countries are common to relate mathematics to "connected values" (ERNEST, 1998, p.45).

4.4.2 Differences

Twenty-five sub-categories (eleven of them are related to isolated thinking and fourteen to connected thinking) for German mathematics teachers and fifteen sub-categories (four of them are related to isolated thinking and eleven to connected thinking) for Turkish mathematics teachers emerged from the interviews. This showed that German mathematics teachers indicated much more values than did Turkish mathematics teachers for both of the categories. In addition, German mathematics teachers responded to be grouped into sub-categories, seventeen of which were different from Turkish Mathematics teachers. At the same time, Turkish mathematics teachers responded to be grouped into the sub-categories, seven of which were different from German mathematics teachers. As the differences was examined in groups, the sub-categories of culture, game, creativity and historical background to be considered under the category of connected thinking attracted the attention. The values crossing them, respectively, are*:* communication, fun, creativity, process. It is important to note that the sub-categories of culture and historical background shows "math is cultural heritage (NCTM, 2000), "cultural phenomenon" (PREDIGER, 2001, p.23) and "a pan-human phenomenon" (BISHOP, 1988, p.180). The following statement taken from the German mathematic education curriculum supports this idea (RAHMENPLAN GRUNDSCHULE

MATHEMATIK – RGM, 2004):

> Mathematical concepts and methods are developed through the questions and problems related to social and practical conditions within the historical process. Mathematics is not a science which is matured yet. It requires an alive and imaginative action. […]Teachers actualize their instructions with individual, community and cultural events (actualize) (p.17-24).

Furthermore, the values of fun and creativity emerging in the present study are also emphasized in German mathematics curriculum (see RAHMENLEHRPLAN FÜR DIE SEKUNDARSTUFE I – RSS, 2006, p.9). On the other hand, the sub-categories of generalization, regularly, systematic, cognition, structural, abstract, accurate, and precise pertaining to the category of "isolated thinking" only emerged from the interview with German mathematics teachers. It can be said that these sub-categories mainly correspond to the value of rationalism. In other words, comparing with Turkish mathematics teachers, German mathematics teachers indicated more alternative thinking related to the value of rationalism. In this case, it is emphasized that "mathematics is a formally structured science" (RGM, 2004, p.17) in German mathematics curriculum. In addition, this may also refer that German mathematics teachers have higher level cognitive process skills on thinking on a problem, conservation with others, mentally calculation of a problem than Turkish mathematics teachers (SCHLOGLMANN, 2008). Furthermore, even though there are Turkish equivalent of these sub-categories expressed by German mathematics teachers, the sub-categories were not mentioned by Turkish mathematics teachers. The sub-categories were also examined in the scope of teachers' concept images regarded as isolated thinking. According to Tall and Vinner (1981), the concept image is "[…] the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (p.152). This refers that Turkish mathematics teachers' mathematics concept images of "isolated thinking" was observed to be limited and narrow.

The sub-categories of intelligence indicator, money, calculation, and a path to God emerging from the interviews with Turkish mathematics teachers are interesting. The values crossing them, respectively, are: tool, tool, rationalism, and tool. The sub-categories of intelligence, indicator and money can be grouped under that societal level of Bishop (1988). According to Bishop (1988), mathematics education is a contest area in order to get academic advantages and seek for various educational opportunities in the society. In Turkey, each individual who would like to receive higher education and to posses a good job should pass the university entrance exam taken after high school education. Mathematics is one of the most important subjects in these exams. For this reason, in order to get success in the subject of mathematics, many students take private mathematics lessons after school. This provides extra income for the mathematics teachers. The sub-category of calculation is also observed to be different in both teacher groups. The studies on seeing mathematics as calculation and computation are also available in the literature (YOUNG-LOVERIDGE; TAYLOR; SHARMA; HAWERA, 2006). Calculation, calculation of area length of the plane figures, calculation with the calculator, calculation of probability and percentages are also emphasized in different learning domains in Turkish mathematics education curricula (MEB, 2009a, 2009b). The sub-category of "a path to God" can be considered to be within the category of individual level of Bishop (1988), and this refers that mathematics is seen as a tool. Then, learners' values and purposes pertaining to mathematics and mathematical thinking may differentiate (BISHOP, 1988). This sub-category may also be applicable to German mathematics teachers. However, in this study, it is believed that a case emerged specific to the city (Berlin) where the study was undertaken. The quotation taken from the interview with one of German mathematics teachers supported this claim

> G5: since there is multiculturalism in German schools, theological and cultural values in mathematics classes are not very important here (in Berlin). But, if you conduct an interview with a teacher in a Catholic school or a teacher working in the state of Bavyera, you will see the differences in this subject (values).

5 Discussion

In the present study, Turkish and German mathematics teachers' views on why math is valuable were investigated, and the mathematical values behind their views were explored. The findings revealed that the mathematical values of mathematics teachers in both countries can be grouped into two categories such as isolated thinking and connected thinking. First of these categories included values that were associated with mathematics as a discipline whereas the other category included values that were associated with the pedagogy of mathematics

(BISHOP, 1988). The results of the study presented that the values of usefulness, rationalism, reasoning, communication, flexibility, aesthetics, tool, and applicability were emphasized by the teachers in both groups. In the other study undertaken with mathematics teachers in a different culture (e.g., VAMP in Australia), similar results were also reported (BISHOP; CLARKSON; FITZSIMONS; SEAH, 2001). This means that mathematics teachers in different cultures hold common approaches with regard to the values related to scientific disciplines of mathematics (e.g., rationalism) (ATWEH; SEAH, 2008). At the same time, this situation is in line with the Platonist view toward mathematics. On the other hand, the common values (mostly related to the pedagogy of mathematics) indicated by both group of teachers are already integrated into the mathematics curricula of both countries (see MEB, 2009a; 2009b; RSS, 2006). This can be considered within the institutional values of Bishop (1988). Institutional values affect the course textbooks and curricula (CLARKSON; BISHOP; SEAH, 2010). On the other hand, in Turkish primary and secondary school curricula (MEB, 2005, 2009a), even though mathematical skills related to *problem solving,* connections, communication and reasoning, and historical development of mathematics and creativity are emphasized, it was observed in the present study that Turkish mathematics teachers indicated only the mathematics values of communication and reasoning. In addition, Turkish mathematics teachers related the mathematics more with connected thinking than with isolated thinking. It is thought that this is due to implementing constructivist approaches since 1997 in Faculties of Education and 2004 in primary and secondary. Besides, the studies conducted with prospective mathematics teachers supported this view (DEDE, 2009; DURMUŞ; BIÇAK, 2006). Furthermore, the following statements taken from Turkish primary mathematics curriculum summarize this situation (MEB, 2009b)

> Mathematics education provides the individuals with knowledge and skills to understand the physical world and social interaction. It earns a language and systematic which help analyze, explain and guess various experiences and solve the problem. Furthermore, it facilitates creative thinking and aesthetic development. In addition, it fastens to develop reasoning skills of individuals while creating an environment where various mathematics cases are examined (p.7).

On the other hand, in the present study, it was found that German mathematics teachers indicated much more approaches in both categories than Turkish mathematics teachers. In addition, German mathematics teachers generally related the mathematics with both isolated thinking and connected thinking. As the literature is examined, it is observed that German mathematics teachers' mathematical 33world views were grouped into four categories such as the aspect of formalism (F), tool or schematic orientation aspect (S), the aspect of process (P), and the aspect of application (A). F and S pertains to the statics aspect of mathematics whereas P and A pertains to the dynamic aspect of mathematics (GRIGUTSCH; RAATZ; TÖRNER, 1998; TÖRNER, 1997). In other words, F and S are considered to be related to Platonism, one of two key philosophy of mathematics, whereas P and A to be related to social constructivism (LANCASTER, 2006). Considering the findings of the present study, F and S can be considered to be within the category of "isolated thinking", and P and A to be within the category of "connected thinking". Törner (1997) reported that there was a significant partial correlation among these four global dimensions. According to this report, there was a high positive correlation between F and P (they represent aspect of the static view of mathematics); a significant negative correlation of P with F and S respectively; and significant correlation between A and P (they represent aspect of the dynamic view of mathematics). Grigutsch et al. (1998) reported that most of German mathematics teachers who attended a teacher training had process- oriented (P) and application-oriented (A) beliefs. Contrary to Grigutsch, Raatz and Törner (1998), Kaiser (2006) found that German mathematics teachers had scheme (tool) oriented and formalism-oriented beliefs. Maass (2009) determined two types of German mathematics teachers holding statics and dynamic views of mathematics (e.g., transmission teacher and learning process teacher). The findings of the present study are parallel to the previous findings.

The other finding obtained from the present study was that some differences were observed between both groups of teachers with regard to the sub-categories associated with the pedagogy of mathematics, and the corresponding values. According to this, different from Turkish mathematics teachers, German mathematics teachers put more emphasis on the sub-categories of culture, game, creativity, and historical background sub-categories and the corresponding values of communication, fun, creativity and process. On the other hand, Turkish mathematics teachers put more emphasis on the sub-categories of intelligence indicator, money, and a path to God. According to these sub-categories, mathematics is seen as tool in general manner. These results obtained from teachers in different cultural background are important to show that mathematical studies differentiate across time, culture and society (LANCASTER, 2006), cultural difference is a factor that have an impact on mathematics teaching (SAM, 2003), different cultures hold different values (BISHOP et al., 2000) and mathematical values differentiate accordingly (SEAH; BISHOP, 1999, 2000).

6 Implications for Future Studies

The most important aim of mathematics education in many of the countries is an enhancement of pedagogical effectiveness. To attain this aim, teachers are important factors to promote egalitarian classrooms and to address long-standing problems of underachieved students (WALSHAW, 2010). Teachers' values affect their decision making to some degree (BISHOP et al., 2001; FASHEH, 1982). To Bishop (1999) "[…] values in mathematics education are deep affective qualities that education fosters through the school subject of mathematics." (p.2). Chin (2006) also says values are part of educational processes in all levels and one of the important parts of the affective, cognitive, and conative environment of mathematics teaching. Therefore, not only mathematical knowledge but also mathematical values are transmitted to students in mathematics teaching. Therefore, it is important for teachers to be aware of the values they have and develop an awareness of values and value preferences toward teaching. Furthermore values are not always positive. Hence, some values that teachers hold may well be quite detrimental to the teaching of mathematics that they undertake, as judged by others, although probably not judged in this way by the teachers themselves (HILL, 1991). Therefore, one of the unique contributions of this current study is to situate the discussion of mathematics teachers' values in two different cultural contexts.

At the end of the present study, similar and different mathematical values that mathematics teachers in both cultures held were determined and then the reasons behind these values were discussed. Aforementioned, this study is limited with the teachers' views to the question of "why mathematics is valuable?" in two different cultures. For this reason, determining teachers' mathematical values in different cultures for mathematical content (e.g., planning, assessment, teaching methods, and decision-making) is an important subject. With this way, more information on teachers' mathematical values can be collected. In addition, this may contribute to the literature on socio-cultural dimensions of mathematics education which groups the values of mathematical thinking into five levels such as cultural level, societal level, institutional level, pedagogical level, and individual level (BISHOP, 1988). Besides, it is standing to study in future studies that what are the reasons that Turkish mathematics teachers' concept images regarded to "isolated thinking" aspect of mathematics are so narrow and limited. It is also needed to study why Turkish mathematics teachers put more emphasis on "connected thinking" aspect of mathematics. For example, what is the effect of newly developed primary and secondary school curriculum on these results? What is the effect of mathematics teacher education programme? It is known that values are hardly changed when compared to beliefs and attitudes (SEAH, 2003), thus, the change of values may also be a direction of future research. On the other hand, it is also a standing problem that which values can be transferred to migrated students with foreign national in a multicultural society, e.g. Germany, and how. For this reason, it had been expected that German mathematics teachers' views included the values of social justice, equity, multiculturalism, and integration. However, German mathematics teacher did not indicate any views related to these values. Investigation of the reasons of this situation may be one of the future research topics.

7 Acknowledgements

I would like to thank all colleagues and mathematics teachers who contributed to this study. Particular thanks to Prof. Dr. Uwe Gellert, Freie Universität Berlin, Berlin. Finally, I am grateful to the Alexander von Humboldt Stiftung (AvH) for funding this research project. The opinions expressed in this article are my own responsibility and they do not necessarily reflect the views of the AvH.

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Submetido em Março de 2012.
Aprovado em Abril de 2012.