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HISTORY OF MATHEMATICS IN DIDACTIC SEQUENCES IN TEACHER'S INITIAL FORMATION

LA HISTORIA DE LA MATEMÁTICA EN SECUENCIAS DIDÁCTICAS EN LA FORMACIÓN INICIAL DE PROFESORES

A HISTÓRIA DA MATEMÁTICA EM SEQUÊNCIAS DIDÁTICAS NA FORMAÇÃO INICIAL DE PROFESSORES

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Abstract

This paper presents results of a case study accomplished in the discipline of Practice of Teaching of Mathematics, of the course of Mathematics of the Federal University of Integration of Latino America (UNILA). Ponte (1994) affirms that a case study is an empiric

investigation whose results are presented, most of the time, in descriptive form. However, it can has an analytical form if it interrogates a situation and to confront it with other known situation. The case study presented in this paper is a qualitative research. In it we analyzed contributions of the history of the mathematics and history of mathematics education for the teachers' initial formation. The data was collected with interview, audios of dialogues concerning the activities among the professor and student, the student's notebook, a video-class produced by him, and professor's annotations. This study of case shows that the history can contribute to the construction of mathematical concepts and for the reflection about mathematics teaching and bring news results when it is confronted to others researches that was made by members of our research group.

Keywords: History. Mathematics. Teacher's Formation.

Resumen

En ese texto presentamos los resultados de un estudio de caso llevado a cabo en la disciplina de la Práctica de la Enseñanza de Matemática, del curso de Matemática de la Universidad Federal da la Integración Latino Americana (UNILA). De acuerdo con Ponte (1994), un estudio de caso es una investigación empírica cuyos resultados son presentados, la mayor parte del tiempo, en la forma descriptiva, pero puede ser también analítica si interroga la situación y la confronta con otra ya conocida. El estudio de caso aquí presentado es de carácter cualitativo. En él analizamos las contribuciones de la historia de la matemática y de la historia de la educación matemática para la formación inicial de profesores. Como instrumentos de la recolección de los datos, usamos entrevista, grabaciones en el audio de diálogos entre la profesora de la disciplina y el alumno con respecto a las actividades, el cuaderno del estudiante, un video - clase producida por él, y las notas en el diario de campo de la profesora. Ese estudio de caso mostró que la historia puede colaborar para la construcción de conceptos matemáticos y para la reflexión sobre la enseñanza de matemática y agrega nuevos resultados en relación a otras investigaciones desarrolladas por los miembros del grupo de investigaciones al cual nosotras pertenecemos.

***Palabras-clave**: Historia. Matemática. Formación de Profesores*

Resumo

Nesse artigo apresentaremos os resultados de um estudo de caso realizado na disciplina de Prática de Ensino de Matemática, do curso de Matemática da Universidade Federal da Integração Latino Americana (UNILA) . Ponte (1994) afirma que um estudo de caso é uma investigação empírica cujos resultados são apresentados, na maioria das vezes, em forma descritiva, porém ela pode ter forma analítica se interrogar a situação e confrontá-la com outras já conhecidas. O estudo de caso apresentado nesse artigo é de caráter qualitativo. Nele, analisamos contribuições da história da matemática e a da educação matemática para a formação inicial de professores. Como instrumentos de coleta de dados, utilizamos entrevista, gravações em áudio de diálogos entre a professora da disciplina e do licenciando acerca das atividades, o caderno do aluno, um vídeo-aula produzido por ele, e anotações da professora em diário de campo. Esse estudo de caso explicitou que a história pode contribuir para a construção de conceitos matemáticos e para a reflexão sobre o ensino de matemática e traz novos resultados quando comparada a outras realizadas por membros do grupo de pesquisa ao qual pertencemos.

Palavras-chave: *História. Matemática. Formação de Professores.*

1 Introduction

Although it was already present in the statutes of the Portuguese University in 1772 (cf. FERREIRA, RICH, 2001), the proposal of insertion of History in the education of Mathematics teachers has been repeatedly made since the late 19th century, as pointed out by Dejić and Mihajlović (2014). However, there is no consensus on how such an insertion would occur. From a review of proposals from texts by different authors, in his doctoral thesis Antonio Miguel categorized several ways in which History could be used in Mathematics classes. According to Miguel (1993), it could be:

1) **Source of appropriate methods of teaching Mathematics**: the authors who support this point of view believe that through History it is possible for teachers to choose appropriate and thought-provoking methods for approaching mathematical content.

2) **Instrument of epistemological awareness:** it is argued that History can play a role in raising awareness of the difficulties that ancient thinkers had in building a given mathematical concept.

3) **A unifying and ethical-axiological instrument:** History would be able to show the transformation process the concepts of Mathematics underwent. Therefore, History would have a demystifying function since the logical form – supposedly harmonious and linear as this discipline is generally seen in regular Mathematics courses – does not match the way in which the content was historically produced.

4) **Source of motivation**: History would arouse students' interest in mathematics classes. Miguel criticizes such a way of understanding History in teaching because he disregards that motivation is something that is not imposed from situations external to people.

5) **Guide to the philosophical debate on mathematical knowledge:** History could show which epistemology would unify a teaching method and deeper Mathematics.

6) **Explanation tool of whys and as a source of teaching objectives**: History could be used as an explanation tool of the whys of concepts and procedures. Naturalized subjects could be questioned in Mathematics teaching, seeking to understand why that theme would be the way it is taught and not otherwise.

7) **Instrument of formalization of concepts**: History would be a means to know different ways of formalization of the same concept and they would serve as object of teaching and learning.

8) **Instrument of cultural rescue:** History would serve to overcome the mathematical knowledge of the colonized who were submerged by the culture imposed by the colonizers. Paulus Guerdes' work on rescuing the mathematical knowledge used to make utensils in Mozambique is an example of this category found in Miguel's thesis (1993).

More recently, Jankvist, Mosvold and Clark (2016) and Lawrence (2009) analyzed the insertion of History in the education of Mathematics teachers from the reference of the teachers' specialized knowledge. Lawrence (2009) concluded that in the context of that teacher group research, History contributed to creating collaboration between them and a creative conceptual panorama for their classes. Jankvist, Mosvold and Clark (2016) reported having asked their student teachers to select from themes of the History of Mathematics and

to prepare Mathematics lesson plans for elementary school. These authors concluded this procedure contributes to building professional knowledge by the student teachers.

In our opinion, History can help teachers to deepen their knowledge about Mathematics and the teaching of this school subject. We agree with Brito and Carvalho (2009), when they state that teachers, besides knowing rules and demonstrating mathematical theorems and algorithms, should also

> be able to relate different fields of this knowledge, reflect on the foundations of Mathematics, perceive its internal dynamism and relations with other fields of knowledge, move through the different systems of representation record and, mainly, understand mathematical knowledge as a knowledge that poses problems and not only solutions. (BRITO, CARVALHO, 2009, p. 16).

In our practice as teacher educators, we have developed didactic sequences in order to achieve the goals presented by the authors mentioned above. A didactic sequence

> is composed of several activities linked to questions, attitudes, procedures and actions that students perform with the teacher's mediation. The activities that are part of the sequence are ordered to deepen the studied theme and are varied in terms of strategy: readings, dialogued lesson, computer simulations, experiments, etc. (MANTOVANI, 2015, p. 17).

Our didactic sequence was composed as follows: resolution and analysis of teaching activities that use History to develop mathematical concepts, analysis of historic content present in a high school Mathematics textbook, production of a seminar to present the results obtained and production of a video.

In the preparation of the teaching activities, we considered first which previous knowledge the students have about the theme to be approached and which difficulties have been historically placed in their learning. Following, we conducted historical research, both in primary and secondary sources, on how that theme developed over time, what problems in Mathematics and other contexts led to its emergence, what forms of recording were used to represent it, what social applications it had, what relations it has with other branches of Mathematics, as it was taught at different times. From these data, we create situations that are not necessarily the same problems found in the History of Mathematics and its teaching, but

that allow students to question the knowledge they already have about the theme, to show themselves their doubts and to build new knowledge in the interaction with the teacher and/or other colleagues. We want to emphasize that in this process we address not only conceptual and procedural aspects of Mathematics, but also axiological and teaching issues. In this sense, we seek to collaborate with a significant construction, by student teachers, of mathematical knowledge and on the school teaching of such subject.

Nunes, Almouloud and Guerra (2010) defend the elaboration of activities that consider the historical context for meaningful building of mathematical concepts. These authors propose

> a conjunction between the significant learning of mathematical concepts and their historical trajectory, highlighting the need to first work with activities that put students in contact with the building of mathematical ideas. We postulate that one of the forms is the historical investigations that aim at the epistemological construction of concepts. (NUNES, ALMOULOUD, GUERRA, 2010, p. 538).

In this article we will report our experience of inserting History in the education of Mathematics teachers. This is a case study conducted in the Mathematics Teaching Practice subject of the 7th semester of the Mathematics-Licentiate Degree course of the Federal University of Latin American Integration (UNILA). This subject has 68 semester hours and is approached in the form of Seminars. Located in the city of Foz do Iguaçu, Paraná, UNILA offers the Mathematics Licentiate Degree course, in the evening period, in the Itaipu Technological Park unit.

UNILA was created by Law No. 12189/2010, which establishes in article 2 paragraph 1 "that it will[...] act in the border regions, with the vocation for academic exchange and joint cooperation with Mercosur members and other Latin American countries". Such action constitutes UNILA as a different institution of higher education, with an international vocation by enabling the conditions of participation of Latin Americans and Caribbeans, for the academic formation aiming at the integration of the countries of Latin America and the Caribbean.

The term “Latin American Integration”, recurring in UNILA documents, according to its Pedagogical Project, includes all countries of the American continent that speak Spanish,

Portuguese or French, as well as other Latin languages. This highlights the bilingual principle at the University, with Portuguese and Spanish.

Headquartered in the city of Foz do Iguaçu, UNILA is strategically located in a trinational region bordering Paraguay and Argentina, with a multicultural, multilingual and economic geographical diversity. Some aspects that favor the promotion of the university principles are interdisciplinarity, interculturality, bilingualism and multilingualism, solidarity integration and democratic management, provided for in the Institutional Development Plan (PDI).

Thus, the courses offered by UNILA should be:

> [...] in areas of mutual interest of the Latin American countries, especially the members of Mercosur, with emphasis on themes involving exploitation of natural resources and transboundary biodiversity, regional social and linguistic studies, international relations and other areas considered strategic for regional development and integration. (BRASIL, 2010, art. 2nd, 2nd paragraph)

There is a Common Cycle of Studies in the courses with lessons on Latin American History; Methodology and Languages – Portuguese for Spanish speakers and Spanish for Brazilians. It lasts three semesters and is offered in parallel to the specific subjects of each course.

In 2014 the Mathematics Licentiate Degree course was created, for the evening period, lasting five years with semester subjects. The insertion of this course in an interdisciplinary, intercultural and bilingual university located in a trinational multilingual region of – autochthonous, allochthonous and frontier languages – provides access to a multicultural experience in the initial education of Mathematics teachers. In this sense, the education of this teacher is articulated with multidisciplinary knowledge based on the ability to analyze problems from the perspective of various cultures involved.

The UNILA Mathematics course was offered for the first time in the first semester of 2018, the subject of Mathematics Teaching Practice 4, which has a set of three other Mathematics Teaching Practices subjects as a prerequisite. Only one student was enrolled in the course. The said student joined the first class and was the only one left in a class that

almost completely evaded. Therefore, we were challenged to teach a single student in at least two subjects: Teaching Practices and Supervised Internship.

We asked the student if we could conduct research on what mathematical knowledge and its teaching would be constituted in the process of development of the subjects already mentioned. The student agreed with our proposal and, thus, this research was configured as a case study.

According to Ponte (1994), a case study is an empirical investigation strongly descriptive in nature, but it does not need to stick to the description because it can have analytical scope when interrogating the situation and confronting it with other situations already known. In it, the investigator has no control over events and therefore needs to be open to possible surprises that may emerge. Such a study may have a qualitative approach or be of a quali-quanti mixed nature. According to Ponte (1994)

> it should also be noted that case studies may be used for purposes other than research. They are used, for example, for teaching, a very common practice in Law and Medicine that is also beginning to be applied in teacher education (Shulman, 1992). Used for this purpose they need not be very detailed in their methodological procedures – they must be illustrative and strongly evocative to the intended audience. (PONTE, 1994, p. 6)

Here, we chose a qualitative approach. The data consisted of the following instruments: answers to a questionnaire proposed on the first day of class, student records in solving activities with historical-mathematical problems, audio recordings of dialogues on activities, student's notebook, student's report on analysis made about the insertion of History in the textbook, audio of the final seminar of the course, a video-lesson produced by him at the end of the process and the notes of the teacher educator in a field diary. The questionnaire consisted of the following questions: 1) What do you understand as History of Mathematics and History of Mathematical Education? 2) Do you consider that the History of Mathematical Education is relevant to the teaching of Mathematics? Why? 3) What school experiences have you had with History of Mathematics in the classroom? Describe them. 4) In which moments of your Mathematics teacher education did you have discussions about the History of Mathematics Education? 5) What would you like to learn in the Mathematics course in the 'History of Mathematics' subject? 6) Are you satisfied with how this subject appears in the Licentiate Degree in Mathematics curriculum?

2 Once upon a time there was a student, a History and a Mathematics

In the second half of 2017 we had agreed to act as one of the centers of the interinstitutional research group 'History, Philosophy and Mathematical Education' (HIFEM) to conduct research on the potential of History in the education of Mathematics teachers. Therefore, at the beginning of the first semester of 2018, we were surprised and disappointed that there was only one student[1] enrolled in the subject. What should be done? In conversation, the two authors of this article agreed that research in this specific situation could add something meaningful to the investigation that was already underway in other group members' workplaces.

However, we realized that due to its specificity, the proposal to be made to the student would need to have some variations in relation to those being made with groups of teachers in other places. In these, the teachers decided together which historical theme to be studied, divided study tasks proposed by themselves and produced texts from these collective studies. The chosen theme need not be related to teaching, but we noted that questions about education in general and specially Mathematics teaching were always present.

In the situation with the only student, because it is a subject of Teaching Practice, we understand that the activities should have the teaching of Mathematics as a principle. We assumed that his previous contact with the History of Mathematics would have been through textbooks in which small historical facts are told at the beginning or end of a unit, without connection to the construction of concepts. This hypothesis was confirmed in the answers of the initial questionnaire in which the student stated:

> In school I had little contact with the History of Mathematics, most of them were historical quotes from who was Pythagoras, Descartes. The few historical experiences in the classroom were when the teacher introduced discovery of counting, natural numbers, early representations of numbers, etc. As well as an activity of copying the historical knowledge of Plato's polyhedra, along with a brief explanation of who Plato was, taken from the textbook (Questionnaire answer, 05/03/2018).

[1] The student's name will not be mentioned at his option.

The student's response is in line with Dejić and Mihajlović (2014). In research conducted between 2012 and 2013 with 112 Mathematics teachers in Serbia, the authors indicate that 80% of these teachers use some History of Mathematics in their classes, and half have only textbooks as sources.

Therefore, we consider that the student should be in contact with teaching activities in which History and Mathematics are inextricably linked, unlike what is found in textbooks. As it is not simple to elaborate activities with this perspective, we chose, at first, to bring activities that we had elaborated and ask the student to choose some to analyze.

On the first class, we talked to the student and presented the proposal that was accepted by him. In that first contact, the student was introspective, shy and had difficulty expressing himself orally. Perhaps being the only one enrolled, in close and direct contact with the teacher, was not a comfortable situation for him. During the semester, he showed responsibility for his own learning and was committed to his studies.

We suggested that he chose some of the activities prepared by us about the History of Mathematics and its teaching, perform them and analyze them from a pedagogical point of view. The activities addressed the following teaching topics: logarithms, Geometry History, numbering systems, tangent, sine and conics. His choices fell on the activities of logarithms and tangent, because according to the student they *"*apparently need more elaborate or even more abstract thinking. Actually, it is because I didn't learn them at school" (Questionnaire answer, 05/03/2018).

The tangent activity sought to focus its geometric and algebraic representations, as well as its concepts from the geometric, algebraic and trigonometric aspects, besides emphasizing the concept of tangent involved in the notion of derivative. It started with a problem with tracing a tangent to a spiral by the Archimedes method and followed with Descartes' studies of tangents to a curve. In the end, the activity addressed aspects of the tangent concept involved in derivative calculations. The student stated that his interest in spirals would have been one of the reasons he chose this activity (Questionnaire answer, 05/03/2018). In addition, on the day of his seminar presentation, the student also reported that his choice fell on this theme, as the concept of tangent was not "very important in school. I didn't learn anything like the way the activity does" (Seminar, 06/08/2018).

In his seminar, the student asserted that

> Most things [about tangent] I already knew how to do. I had seen in Calculus to make derivatives because of some definitions, we use that a lot. Well, [the activity] contemplated [the expectation] in the historical perspective, because doing and saying that it is just the way it is is easy, but now, doing and saying why this is like that is totally different (Seminar, 06/08/2018).

Here we observe a criticism of the student in relation to the teaching of school Mathematics that, for the most part, presents definitions and rules and does not explain why. In addition, this excerpt from the seminar brings us to one of the functions that History can perform in Mathematics classes, namely, to denaturalize mathematical knowledge and to explain why concepts and rules are the way they are in the school curriculum now (MIGUEL, 1993; NOBRE, 1996).

The perception of the need to explain definitions, rules and concepts to his future students meant that the teacher student, in his video class, did not address the question of Archimedes' tangent to the spiral, as we observed in the excerpt below:

> Student: From the tangent of [Archimedes]'s spiral – I am so far trying to understand how he [Archimedes] was able to carry the length of the arc to the length of a segment[2]. Like... I could calculate it, but, oh boy! How could he do that? I understand why, but I don't understand how in his time he was able to carry that. So far I keep thinking [...] So much so that in the part of making a video, I just mentioned the normal line with the tangent line. Because really, if I do it will be the easiest way, I'll calculate the length of the arc, by the way, Geogebra gives me this, I transport. But why? How will I explain that? I will not make a video where I will not be able to explain what I did.
>
> Teacher: Do you think History helps to justify?
>
> Student: It does. But I can't understand the context (Seminar, 06/08/2018).

This passage explains that the student is no longer satisfied with a logical-mathematical explanation, but realizes the need for historical explanation and to study the

[2] For further explanation of the construction of the tangent to the spiral we suggest that the reader consult the book Curso de História da Matemática: origens e desenvolvimento do Cálculo. Brasília: Editora da UNB, 1985.

context more deeply to understand how Archimedes would have transferred the measurement of an arc to a segment and thus he can explain this process to his future students. History would be playing a role as an instrument of epistemological awareness (MIGUEL, 1993), since the student posed the question of difficulties that Archimedes might have faced in tracing the tangent to the spiral without the current conceptual and technological resources.

The logarithm activity began by defining them from arithmetic and geometric progressions (AP and GP), found in an early 20th century textbook. It then presented a table that should be filled in, following how, according to Carl Boyer*'s History of Mathematics*, Napier (1550-1617) would have developed his initial logarithm idea. Then it proposed to build a logarithm to the base 10.

According to the student, his choice for the study of logarithms occurred because *"*this was the only theme I had no contact with at school and also because I did not see it structured in this way" (Questionnaire answer, 05/03/2018). Also, in his seminar presentation, he stated that he had learned logarithms at university just to do calculations.

The student reported that "having the first contact with the activity, I had the impression that it would not be a content that would be so hard to understand what was asked of it, because even though I had no contact with logarithms at school, I often use them in calculations" (Resolution of activities, 05/24/2018). Here the student reports his belief that his knowledge of logarithmic calculus rules would be enough for him to solve the proposed questions. However, according to notes in his notebook, even knowing part of the concepts involved, as the questions did not require only knowledge about rules, the student had difficulties in solving them, besides realizing that his knowledge of mathematical language was not enough for him to transcribe the definition of logarithm found in old books to the current notation and rigor. In the process of teacher education, it is important for them to experience and analyze the difficulties that may arise in transforming one form of representation into another, such as from mother tongue to algebraic or from algebraic to geometric, as these occur often in elementary school classes and sometimes make it difficult or even prevents students to access mathematical knowledge. Such transformations were required in this activity.

In his seminar, the student stated that

> First part [of the activity] that would be a bit obvious is to match logarithms and numbers to be logarithmed. Practically simpler, more understandable, it's not like, 'this is the logarithm definition', but if that number raised to something results in that. The activity made it possible to relate the properties of logarithms more broadly than a definition we see today (Seminar, 06/08/2018).

Tzanais and Thomaidis (2000) affirmed that the meaning of a concept, theorem or method is not completely determined by its modern definition and the History of Mathematics may suggest alternative ways of approaching them and a variety of conditions from which they can be understood.

Our data indicate that it was possible for the student to understand the historical context of logarithm creation, because

> you know where the idea came from, how it was developed, what it served for in that historical context of Napier [and for] their other contemporary mathematicians. Interesting!! In fact, his idea is something that at the time was "outside the box", mixing geometry with other areas [...] to find a tool (Seminar, 06/08/2018).

This excerpt shows History playing a role in instructing the teacher to answer the question "what is this for?" often asked by students in classes, because, according to Brito (2007), History can help the teacher to analyze which problems led to the development of mathematical theories, whether they come from this field of knowledge or others, or from practical needs.

As a conclusion of this part of the didactic sequence, the student reveals that he realized that History can be a source of teaching method (MIGUEL, 1993), but he raises some difficulties for this:

> Teachers are supposed to know more than what they teach. They will probably teach in the usual way, but if there is time left they can choose and teach using History as a teaching methodology. At the internship, I realized that all teachers followed the book, but it's a necessity. I see that it is because of lack of time (Seminar, 06/08/2018).

The first difficulty, as shown in the excerpt above, is the teacher's knowledge of History and how to integrate it into Mathematics classes. Such difficulty is also pointed by

several authors whose researches address this subject. Thus, according to Ferreira and Rich (2001), teachers talk about the benefits of such integration, but highlight the little knowledge of how to do it. Brito, Santos and Teixeira (2009) conducted research on the view of undergraduates about the use of History as a methodological resource and, according to these authors, they point out as some obstacles to such use: the lack of knowledge of History of Mathematics on the part of student teachers, the difficulty of access to historical sources, the lack of activities with this methodological approach, and also the little time for class preparation and curriculum development in class.

Time has been an obstacle for teachers to innovate in their classes, because according to Tardif et al (2001),

> the temporal structure of the school organization is extremely embarrassing for teachers, because in some way it constantly pushes them forward, forcing them to repeat this collective and abstract cycle that does not depend on the slowness or speed of students' learning. School time is a social and administrative time imposed on individuals, it is a forced time (TARDIF et al, 2001, p. 42).

The adoption of textbooks has not contributed so that History can be part of the Mathematics classes, as the student observed in his Seminar, in the previously mentioned excerpt, and in the activity of analysis of historical elements in the sophomore high school Mathematics textbook. In his report, the student pointed out some situations found in the book, such as Archimedes' process for determining the circumference length that is later used in an exercise, the context of using matrices in ancient China, and several biographies. He concluded that History was mobilized in the book

> at the introduction or end of a concept or content, and at the end of the chapter as other contexts, where the origin of a specific theme or concept is described here. It also appears in the middle of chapters as reading (brief in general, biographical) and as curiosity (also biographical) (Report, 06/01/2018).

Moreover, in discussion with the professor about this activity, the student teacher stated that the historical aspects evidenced by the textbook would be superficial and would

not add to the students' learning. Such observations triggered by this activity agree with what Ferreira and Rich (2001) state:

> Still another reason why History has not been integrated into school Mathematics concerns the ways, if any, in which textbooks typically address the History of Mathematics, mostly limited to an inclusion of a few historical notes (generally, biographies and curiosities) at the end of each chapter (FERREIRA, RICH, 2001, p. 71).

The video class was an option of the student who made an exposition on tangent mobilizing History and technology – video and Geogebra – as didactic resources. The class followed a lecture model, but indicated the student's concern to explain in detail the knowledge involved in tracing a tangent to a curve.

We want to emphasize that the integration of History in Mathematics classes does not dispense with the use of technology, because to deal with the mathematical and historical complexity of the didactic sequence proposed by us, the student turned to books on the Internet and among them found the early 20th century textbook that had inspired the first activity of logarithms; he watched video lectures by Professor João Bosco Pitombeira of the Pontifical Catholic University of Rio de Janeiro (PUC-RJ) on YouTube about the context of logarithm creation and how they were used by different mathematicians at different times; he researched articles in English and even in Italian to try to understand Archimedes' tangent tracing process. Such actions indicate the student's involvement with the proposed activities. After all this process, the student's knowledge increased not only in relation to the mathematical concepts involved and to the questions about the school teaching of this subject, but also regarding possible sources of study that could be used in his future classes. According to the student, "the activity itself was extremely important, as it showed how the History of Mathematics can develop mathematical knowledge and *its importance for teaching and understanding* them" (Audio Testimony, 06/07/2018, emphasis added).

3 Considerations

The researches carried out by HIFEM have pointed out the potential of History for the education of Mathematics teachers. However, previous researches by this research group had

been done in collaborative groups of teachers, which raised doubts about the positive results obtained by these investigations: were they caused by the historical problematization or by the dynamics of the collaborative group? Without denying this last hypothesis, the case study reported here brought a new element to the discussion on the theme by indicating that even if not performed in a collaborative group, History can contribute to the expansion of teachers' mathematical knowledge and to their questions about the school teaching of Mathematics.

We also noticed that the didactic sequence for the analysis of the insertion of History in the education of the Mathematics teacher, prepared by us, met the assumptions presented in UNILA's PDI, as aspects of knowledge interdisciplinarity were highlighted, and interculturality was mobilized by leading the student to reflect on how and why people in other historical moments and cultures produced and taught Mathematics, as we saw in his questions about Archimedes, Napier, and how logarithms were taught in the early 20th century, as well as favoring multilingualism, as the student has embarked on several languages in his Internet investigations on the themes addressed.

Through this methodology we hope to be able to also collaborate with the solidarity integration of students coming from countries of the triple border and others.

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